MTH 605: Topology I

Practice Assignment VI

1. Let $p : \widetilde{X} \to X$ is a covering map. Show that if $\widetilde{X}$ is path-connected and X is simply-connected, then p is a homeomorphism.

2. Consider the universal covering space $p^2 : \mathbb{R}^2 \to S^1 \times S^1$ of the torus. Under this covering map, find a lifting of the loop $\alpha(m, n) = (e^{m\pi s}, e^{n\pi s})$ on the torus to $\mathbb{R}^2$.

3. Show that if A is a retract of the D^2, then every continuous map $f : A \to A$ has a fixed point.

4. Let $p : \widetilde{X} \to X$ be a covering space with $p^{-1}(x)$ finite for all $x \in X$. Show that $\widetilde{X}$ is compact Hausdorff iff X is compact Hausdorff.

5. Show that if a path-connected, locally path-connected covering space X has a finite fundamental group, then every map $f : X \to S^1$ is nullhomotopic.

6. Consider the map $p \times i : \mathbb{R} \times \mathbb{R}_+ \to S^1 \times \mathbb{R}_+$, where i is the identity map of $\mathbb{R}_+$ and $p : \mathbb{R} \to S^1$ is the universal covering space.

   (i) Show that $p \times i : \mathbb{R} \times \mathbb{R}_+ \to S^1 \times \mathbb{R}_+$ is a covering space.

   (ii) Sketch the paths $f(t) = (2 - t, 0)$, $g(t) = (1 + t) \cos 2\pi t, (1 + t) \sin 2\pi t)$, and $h(t) = f * g$, and also their liftings under the covering space above.

7. Read Lemma 55.3, Corollary 55.4, and Theorem 55.5 from Munkres.

8. Assume that there is no retraction $r : D^{n+1} \to S^n$. Show that every continuous map $f : D^n \to D^n$ has a fixed point. [Hint: Generalise the three results in problem 7 to S^n.